ANALYSIS OF WAVEGUIDES WITH DIELECTRIC INSERTS

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ABSTRACT

The method, which has been introduced in [1] for the analysis of waveguides with metal inserts, is extended to the analysis of waveguides with dielectric inserts. These include shielded dielectric guides, e.g. dielectric image guide and dielectric rod guide, and all shielded planar guiding structures. The method is based on applying the equivalence principle followed by field expansions in terms of the normal modes of the corresponding waveguide without the dielectric insert.

The validity of the method is checked by analyzing a simple dielectric-slab-loaded rectangular waveguide. The method is then applied to the analysis of more complicated structures, e.g. shielded dielectric image guide and shielded dielectric rod guide.

INTRODUCTION

Most of the existing guiding structures can be considered as waveguides with dielectric inserts. Examples are shielded planar guiding structures, e.g. microstrip lines, finlines, and suspended substrate lines, dielectric loaded rectangular and circular waveguides, and shielded dielectric image guides. A unified method for the analysis of such a wide class of structures is not yet available. Such a method is strongly recommended for computer aided design and optimization of complex microwave systems, in which different guiding structures are involved.

The method presented in this paper, which fulfills this requirement, depends on applying the equivalence principle to replace the dielectric insert by electric and magnetic surface currents. The field excited by these currents is then expanded with respect to the eigenmodes of the corresponding empty waveguide. The yet unknown electric and magnetic surface currents are expanded with respect to properly chosen basis functions, and the continuity of the field across the air-dielectric interface(s) is tested by the same functions.

The analysis presented here is an extension of that presented in [1] and [2]. It can be considered a generalized spectral domain (GSD) analysis and has, consequently, all advantages of the conventional spectral domain (SD) technique (see e.g. [3] - [5]).

BASIC FORMULATION

Fig. 1 shows the cross section of a waveguide with a homogeneous dielectric insert. Extending the analysis to multi-insert guides is straight-forward. The direction of propagation, in which the structure is uniform, is taken along the z-axis with corresponding propagation constant β.

Applying the equivalence principle (see e.g. [6]), the electromagnetic field inside the guide is the sum of two complementary partial fields. The first, which will be called the outer field, vanishes identically over the insert region S_0 and is excited by the electric and magnetic surface currents $\underline{J}_S = \hat{\underline{n}} \times \underline{H}$ and $\underline{M}_S = \underline{E} \times \hat{\underline{n}}$, respectively, which reside on the contour C_0 and radiate inside the air-filled guide. The second partial field, which will be called the inner one, vanishes identically over the air region $(S - S_0)$, and is excited by $-\underline{J}_S$ and $-\underline{M}_S$ radiating in a waveguide which is homogeneously filled with the insert material.

Because the structure is inhomogeneously filled, the different eigenmodes are in general hybrid. The axial electric and magnetic fields must then both exist and both divergence-free and curl-free complete sets are necessary for the expansion of the transverse components.

Let $\{h_{zn}\}$ and $\{e_{zn}\}$ be the complete sets of axial magnetic and electric fields which characterize the TE and TM modes, respectively, of the air-filled waveguide (i.e. with the dielectric insert S_0 removed). h_{zn} and e_{zn} are real functions of the transverse coordinates, which correspond to cutoff wave numbers k_{nh} and k_{ne}, respectively, and satisfy the following orthogonality relations [7]:

$$\int_S h_{zn}\, h_{zm}\, dS = P_{nh}\, \delta_{nm},$$

$$\int_S e_{zn}\, e_{zm}\, dS = P_{ne}\, \delta_{nm}. \qquad (1)$$

Let also $\underline{e}^{(o)}$ ($\underline{e}^{(i)}$), $\underline{h}^{(o)}$ ($\underline{h}^{(i)}$), $e_z^{(o)}$ ($e_z^{(i)}$), and $h_z^{(o)}$ ($h_z^{(i)}$) be the outer (inner) transverse electric, transverse magnetic, axial electric, and axial magnetic field, respectively, with the z-dependence $e^{-j\beta z}$ being dropped out. According to the completeness properties of $\{e_{zn}\}$ and $\{h_{zn}\}$, the above components can be expanded as

$$\underline{e}^{(\nu)} = \sum_n \frac{a_n^{(\nu)}}{\sqrt{P_{ne}}}\, \underline{\nabla}_t e_{zn} + j\omega\mu_o \sum_n \frac{b_n^{(\nu)}}{\sqrt{P_{nh}}} \left[\hat{\underline{k}} \times \underline{\nabla}_t h_{zn}\right],$$

$$\underline{h}^{(\nu)} = \sum_n \frac{c_n^{(\nu)}}{\sqrt{P_{nh}}}\, \underline{\nabla}_t h_{zn} + j\omega\varepsilon_o \sum_n \frac{d_n^{(\nu)}}{\sqrt{P_{ne}}} \left[\underline{\nabla}_t e_{zn} \times \hat{\underline{k}}\right],$$

$$e_z^{(\nu)} = \sum_n k_{ne}^2 \frac{f_n^{(\nu)}}{\sqrt{P_{ne}}}\, e_{zn},$$

$$h_z^{(\nu)} = \sum_n k_{nh}^2 \frac{g_n^{(\nu)}}{\sqrt{P_{nh}}}\, h_{zn}, \qquad (2)$$

where ν stands for o or e, $\underline{\nabla}_t$ is the transverse del-operator, and $\hat{\underline{k}}$ is the unit vector in axial direction. The expansion coefficients $a_n^{(\nu)}$, $b_n^{(\nu)}$, $c_n^{(\nu)}$, $d_n^{(\nu)}$, $f_n^{(\nu)}$, and $g_n^{(\nu)}$ are obtained by making use of the orthogonality relations (1) and by integrating Maxwell's equations by parts. This results in expressing these expansion coefficients in terms of contour integrals involving the tangential components of the yet unknown total field at C_0.

APPLICATION OF GALERKIN'S METHOD

The tangential field components at C_0 are next expanded in terms of suitable basis functions

$$e_z = \sum_i V_i^{(1)} \tilde{\xi}_i,$$

$$e_\tau = \sum_i V_i^{(2)} \xi_i,$$

$$h_z = \sum_i I_i^{(1)} \tilde{\eta}_i,$$

$$h_\tau = \sum_i I_i^{(2)} \eta_i, \qquad (3)$$

where $V_i^{(1)}$, $V_i^{(2)}$, $I_i^{(1)}$ and $I_i^{(2)}$ are expansion coefficients and $\tilde{\xi}_i$, ξ_i, $\tilde{\eta}_i$, and η_i are the basis functions. Substituting (3) in the contour integrals, the field expansion coefficients $a_n^{(\nu)}$, $b_n^{(\nu)}$, $c_n^{(\nu)}$, $d_n^{(\nu)}$, $f_n^{(\nu)}$, and $g_n^{(\nu)}$ are obtained as linear combinations of $V_i^{(1)}$, $V_i^{(2)}$, $I_i^{(1)}$, and $I_i^{(2)}$. Equating the outer and inner fields at C_0 and testing the resulting continuity equations by the basis function $\tilde{\xi}_i$, ξ_i, $\tilde{\eta}_i$, and η_i (Galerkin's procedure), a homogeneous system of linear equations is obtained, which must be solved for the propagation constant β and the expansion coefficients $V_i^{(1)}$, $V_i^{(2)}$, $I_i^{(1)}$, and $I_i^{(2)}$. This system can be written in the following matrix form:

$$\begin{bmatrix} [Z^{(11)}] & [T^{(11)}] & [Z^{(12)}] & [T^{(12)}] \\ [T^{(21)}] & [Y^{(21)}] & [T^{(22)}] & [Y^{(22)}] \\ [\tilde{Z}^{(11)}] & [\tilde{T}^{(11)}] & [\tilde{Z}^{(12)}] & [\ 0\ ] \\ [\tilde{T}^{(21)}] & [\tilde{Y}^{(21)}] & [\ 0\ ] & [\tilde{Y}^{(22)}] \end{bmatrix} \begin{bmatrix} \underline{I}^{(1)} \\ \underline{V}^{(1)} \\ \underline{I}^{(2)} \\ \underline{V}^{(2)} \end{bmatrix} = 0, \qquad (4)$$

where $\underline{I}^{(1)}$, $\underline{V}^{(1)}$, $\underline{I}^{(2)}$, and $\underline{V}^{(2)}$ are column vectors with elements $I_i^{(1)}$, $V_i^{(1)}$, $I_i^{(2)}$, and $V_i^{(2)}$, respectively, and the elements of the submatrices of the characteristic matrix are functions of β^2.

APPLICATION TO SHIELDED DIELECTRIC ROD GUIDE

Consider the shielded dielectric rod guide with cross section shown in Fig. 2-a. Due to the symmetry of the guide, it is sufficient to analyze the structure shown in Fig. 2-b, with the two symmetry planes at x = 0 and y = 0 being electric or magnetic walls. For illustration, the case with the two symmetry planes being electric walls has been analyzed. The basis functions in (3) then read

$$\tilde{\xi}_i(\varphi) = \eta_i(\varphi) = \sin 2i\varphi,$$

$$i = 1, 2, \cdots,$$

$$\xi_i(\varphi) = \tilde{\eta}_i(\varphi) = \cos 2i\varphi,$$

$$i = 0, 1, 2, \cdots. \qquad (5)$$

Because the investigated structure is inhomogeneously filled, it can support complex modes [8]. Our computer code was, however, written for looking for real β^2. It is currently being modified in order to look for complex β^2 as well. Table I shows the propagation constants of the first 5 non-complex modes at a frequency lying below the TE_{10} cutoff frequency of the shielding wayeguide. The field distributions of the outer and inner axial fields along the main diagonal are plotted in Fig. 3. All components have step discontinuities at the air-dielectric interface, which are accompanied with the so-called Gibb's effect. Except for the fluctuations due to Gibb's effect, all components vanish where they should. Fig. 4 shows plots for the total axial fields. The fluctuations in the outer and inner fields eliminate each other and continous fields are obtained.

Other structures are currently being investigated (e.g. a shielded dielectric image guide). The results will be presented in the conference.

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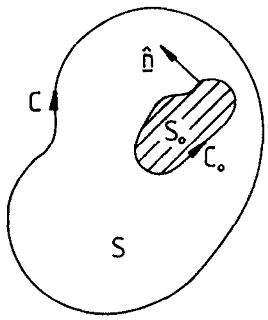

Fig. 1: Cross section of a waveguide with a dielectric insert.

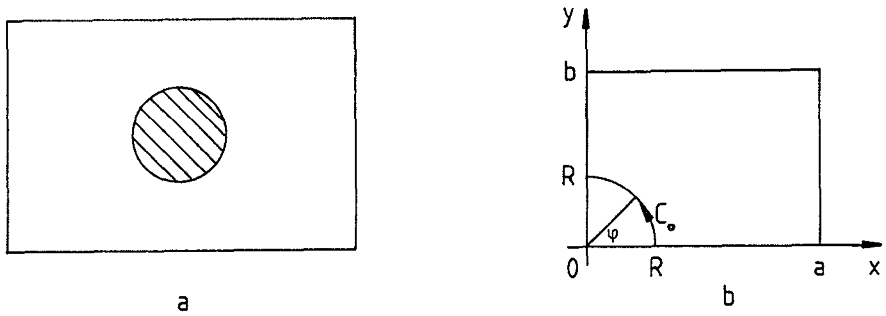

Fig. 2: a) Shielded dielectric rod guide, b) A quarter of the structure.

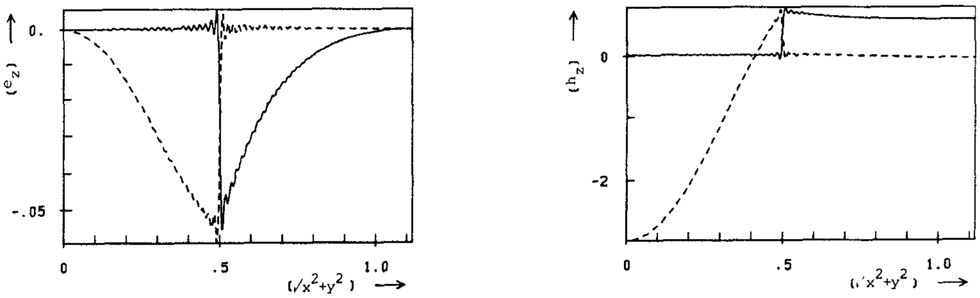

Fig. 3: Axial components of the outer and inner fields, —— : outer, ---- : inner. Parameters: as in Table I.

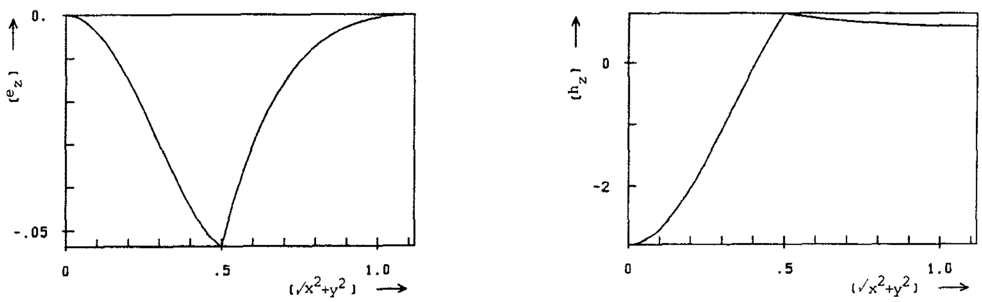

Fig. 4: Total axial electric and magnetic fields. Parameters: as in Table I.

mode	1	2	3	4	5
$(\beta a/\pi)$	+0.9311	-j1.4488	-j1.9577	-j2.5927	-j2.7743

Table I: Normalized propagation constants of the first 5 non-complex modes in the structure of Fig. 2-b. Parameters: $R = b = a/2$, $\varepsilon_r = 5$, $(k_0 a/\pi) = 0.75$.